



**APROXIMACIÓN NUMÉRICA DEL PROBLEMA DE DIRICHLET
PARA LA ECUACIÓN DE POISSON EN EL MEDIO LENTE**

**APROXIMAÇÃO NUMÉRICA DO PROBLEMA DE DIRICHLET
PARA A EQUAÇÃO DE POISSON NO MEIO DA LENTE**

**NUMERICAL APPROXIMATION TO THE DIRICHLET PROBLEM
FOR THE POISSON EQUATION IN HALF LENS DOMAIN**

Autores:

Resumen

✉ **Diego Toala**^{1,*}



✉ **Judith Vanegas**²



¹Maestría en Matemática, Facultad de Posgrado, Universidad Técnica de Manabí, Manabí, Ecuador.

²Vicerrectorado Académico, Universidad Yachay Tech, Urcuquí, Ecuador.

* Autor para correspondencia.

Editor Académico

Juan Carlos Osorio López

Citación sugerida: Toala D., Vanegas J. (2025). NUMERICAL APPROXIMATION TO THE DIRICHLET PROBLEM FOR THE POISSON EQUATION IN HALF LENS DOMAIN. *Revista Bases de la Ciencia*, 10(2), 14-23. DOI: 10.33936/revbasdelaciencia.v10i2.7544

Recibido: 25/05/2025

Aceptado: 20/08/2025

Publicado: 20/08/2025

Este trabajo presenta una solución aproximada del problema de Dirichlet para la ecuación de Poisson en dos dimensiones, en un dominio con geometría de medio lente. A partir de la función de Green específica para este dominio, se obtuvo una solución particular basada en la representación integral del problema. A continuación, se implementó un enfoque numérico en MATLAB para evaluar dicha representación integral. Los resultados numéricos presentados en tablas y gráficos, demuestran el rendimiento del método con métricas del tiempo computacional. Esta solución representa un primer acercamiento a próximas implementaciones numéricas en métodos rápidos como Transformada Rápida de Fourier.

Palabras clave: Problema de frontera de Dirichlet, medio lente, ecuación de Poisson, aproximación numérica.

Abstract

This work presents an approximate solution to the Dirichlet problem for the two dimensional Poisson equation in a half lens domain. Based on the Green's function specific to this domain, a particular solution was derived obtaining the integral representation of the problem. Subsequently, a numerical approach was implemented in MATLAB to evaluate this integral representation. The numerical results, presented in tables and plots, demonstrate the method's performance using metrics such as execution time. This solution represents a first step toward future numerical implementations using fast algorithms, including the Fast Fourier Transform.

Keywords: Dirichlet boundary value problem, half lens domain, Poisson's equation, numerical approximation.

Resumo

Este trabalho apresenta uma solução aproximada para o problema de Dirichlet da equação de Poisson em duas dimensões, em um domínio com geometria de meia lente. A partir da função de Green específica para esse domínio, foi obtida uma solução particular baseada na representação integral do problema. Em seguida, foi implementada uma abordagem numérica no MATLAB para avaliar essa representação integral. Os resultados numéricos, apresentados em tabelas e gráficos, demonstram o desempenho do método com métricas como o tempo de execução. Esta solução representa um primeiro passo para futuras implementações numéricas com algoritmos rápidos, como a Transformada Rápida de Fourier.

Palavras chave: Problema de fronteira de Dirichlet, meia lente, equação de Poisson, aproximação numérica.



1. Introduction

The Poisson equation plays an important role in analyzing physical problems across multiple fields. Although exact and approximate solutions exist for simple geometries like rectangles and disks, solving it in more complex domains such as half lens, remains especially challenging.

Analytical solutions to the Dirichlet problem for the Poisson equation have also been the subject of extensive research in a variety of domains, including the circular ring, infinite horizontal strip of width π , cracked half-plane, doubly connected domains, and lens shaped domain (Cedeño & Vanegas, 2022; Taghizadeh & Mohammadi, 2017; Vaitiakhovich, 2008; Velez Cantos & Vanegas Espinoza, 2022; Vergara Ibarra & Vanegas Espinoza, 2022). These solutions are typically derived by constructing an appropriate Green's function, which is then employed to obtain the solution via an integral representation.

Moreover, approximated solutions to Dirichlet problems for the Poisson equation in lens-like domains are crucial for progress in applied optics, modeling, and the predictive study of light behavior in complex cosmological contexts (Mandelbaum, 2018), as well as in metamaterials research aimed at improving performance in applications like solar photovoltaic systems (Memarian & Eleftheriades, 2013).

In addition, efficient and accurate computation of solutions using Green's function for Poisson equation has been applied in rectangular waveguides and acoustic flows taking advantage of MATLAB's computational capabilities to obtain numerical solutions to complex partial differential equations (Berger & Lasher, 1958; Cogollos y col., 2009; Harwood & Dupère, 2012; Shior y col., 2024).

This paper deals with the Dirichlet boundary value problem for the Poisson equation as the form:

$$w_{zz} = f \quad \text{in } \Omega, \quad w = \gamma \quad \text{on } \partial\Omega, \quad \gamma(m-r) = \gamma(1) = 0, \quad (1)$$

for $f \in L_1(\Omega; \mathbb{C}) \cap C(\Omega; \mathbb{C})$, $\gamma \in C(\partial\Omega; \mathbb{C})$ and w is the solution in the space $C^2(\Omega; \mathbb{C}) \cap C(\overline{\Omega}; \mathbb{C})$.

As for the lens domain Δ (Begehr & Vaitekhovich, 2014), this can be constructed by $\Delta = \mathbb{D} \cap D_m(r)$, where $\mathbb{D} = \{z : |z| < 1\}$ denotes the unit disk and $D_m(r) = \{z : |z - m| < r\}$ represents a disk centered at m with radius r , satisfying $0 < r < 1 < m$, and $r^2 + 1 = m^2$. Consequently, the half lens Ω is defined by $\Omega = \{z \in \Delta : \text{Im } z > 0\}$ which is depicted in Figure 1.

Note that the domain Ω has three distinct boundaries defined by $z \in (m-r, 1)$, that is, $z = \bar{z}$. For $z \in \partial\Omega_{\mathbb{D}}$, that is, $z\bar{z} = 1$ and $z \in \partial\Omega_{D_m}$, that is, $|z - m| = r$.

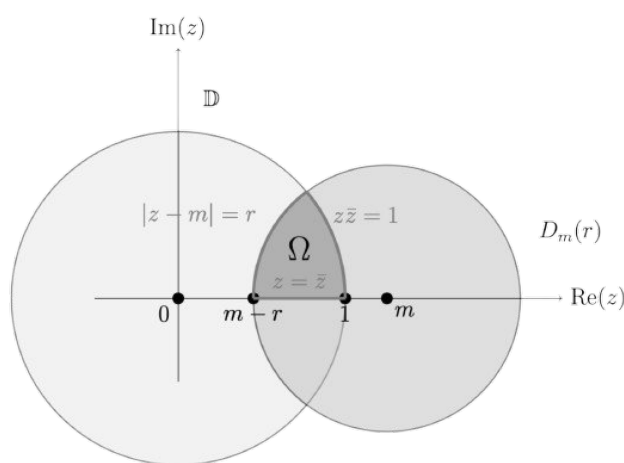


Figura 1. The half lens Ω .

Dirichlet problems for the Poisson equation represent a significant class of elliptic partial differential equations problems that have been extensively studied. Traditionally, solving the Dirichlet boundary value problems for the Poisson equation relies on the use of integral representation equation involving Green's functions.

2. Mathematical Foundations

2.1. Analytical solution to the Dirichlet problem for Poisson equation in half lens

Teorema 2.1. *The Dirichlet problem*

$$\omega_{zz} = f \text{ in } \Omega, \quad \omega = \gamma \text{ on } \partial\Omega, \quad \gamma(m-r) = \gamma(1) = 0, \quad (2)$$

has a unique solution, given specified functions $f \in C(\Omega, \mathbb{C})$ and $\gamma \in C(\partial\Omega, \mathbb{C})$. The solution is explicitly provided by

$$\begin{aligned} \omega(z) = & \frac{1}{2\pi i} \int_{m-r}^1 \gamma(t) \left[\frac{z - \bar{z}}{|t - z|^2} - \frac{z - \bar{z}}{|zt - 1|^2} \right. \\ & \left. + \frac{r^2(z - \bar{z})}{|t(1 - mz) - (m - z)|^2} - \frac{r^2(z - \bar{z})}{|t(m - z) - (1 - mz)|^2} \right] dt \\ & + \frac{1}{2\pi i} \int_{\partial\Omega_D} \gamma(\zeta) \left[\frac{1 - |z|^2}{|\zeta - z|^2} - \frac{1 - |z|^2}{|\bar{\zeta} - z|^2} \right. \\ & \left. - \frac{r^2(1 - |z|^2)}{|\zeta(1 - mz) - (m - z)|^2} + \frac{r^2(1 - |z|^2)}{|\bar{\zeta}(1 - mz) - (m - z)|^2} \right] \frac{d\zeta}{\zeta} \\ & + \frac{1}{2\pi i} \int_{\partial\Omega_{D_m}} \gamma(\zeta) \left[\frac{r^2 - |z - m|^2}{|\zeta - z|^2} - \frac{r^2 - |z - m|^2}{|\bar{\zeta} - z|^2} + \frac{r^2|\zeta - m|^2}{|\zeta - m|^2} \right] \frac{d\zeta}{\zeta - m} \\ & - \frac{1}{\pi} \int_{\Omega} f(\zeta) G_1(z, \zeta) d\zeta d\eta, \end{aligned} \quad (3)$$

where $G_1(z, \zeta)$ is the harmonic Green function for the half lens Ω given by:

$$\log \left| \frac{(\bar{\zeta} - z)(1 - z\bar{\zeta})(\bar{\zeta}(1 - mz) - (m - z))(\bar{\zeta}(m - z) - (1 - mz))}{(\zeta - z)(1 - z\zeta)(\zeta(1 - mz) - (m - z))(\zeta(m - z) - (1 - mz))} \right|^2. \quad (4)$$

For $z \in (m-r, 1)$, that is, $z = \bar{z}$, implies that

$$\partial_{v_z} G_1(z, \zeta) = -i(\partial_z - \partial_{\bar{z}}) G_1(z, \zeta) \quad (5)$$

$$= 4Im \left[\frac{1}{\zeta - z} + \frac{\zeta}{1 - z\zeta} + \frac{m\zeta - 1}{\zeta(1 - mz)(m - z)} + \frac{\zeta - m}{\zeta(m - z) - (1 - mz)} \right],$$

for $z \in \partial\Omega_D$, that is, $z\bar{z} = 1$, it follows that

$$\partial_{v_z} G_1(z, \zeta) = (z\partial_z + \bar{z}\partial_{\bar{z}}) G_1(z, \zeta) \quad (6)$$

$$= 4Re \left[\frac{z}{\zeta - z} + \frac{1}{1 - z\zeta} + \frac{z(\zeta - m)}{\zeta(m - z) - (1 - mz)} + \frac{(m - \zeta)}{\zeta(mz - 1) - (z - m)} \right],$$

and for $z \in \partial\Omega_{D_m}$, that is, $|z - m| = r$, thus

$$\begin{aligned} \partial_{v_z} G_1(z, \zeta) = & \left(\left(\frac{z - m}{r} \right) \partial_z + \left(\frac{\bar{z} - m}{r} \right) \partial_{\bar{z}} \right) G_1(z, \zeta) \\ = & \frac{1}{r} 4Re \left[\frac{z - m}{\zeta - z} + \frac{\zeta(z - m)}{1 - z\zeta} + \frac{\zeta r^2}{\zeta(mz - 1) - (z - m)} + \frac{r^2}{\zeta(m - z) - (1 - mz)} \right]. \end{aligned} \quad (7)$$



Given a specific f and γ , a particular solution can be obtained using the integral representation in Equation (3) to evaluate numerical results against an exact solution. Possible singularities arise when any of the following terms in the denominator of the Green function in Equation (4) becomes zero:

1. $(\zeta - z)$:

- Singular if $\zeta = z$.

2. $(1 - z\zeta)$:

- Singular if $z\zeta = 1$, or equivalently $\zeta = \frac{1}{z}$.

3. $(\zeta(1 - mz) - (m - z))$:

- Singular if:

$$\zeta = \frac{m - z}{1 - mz}.$$

4. $(\zeta(m - z) - (1 - mz))$:

- Singular if:

$$\zeta = \frac{1 - mz}{m - z}.$$

These singularities may influence the evaluation depending on the location of z . When z lies within the domain, singularities 1 and 2 may arise, whereas on the boundary, singularities 3 and 4 may occur. These singular points must be avoided during the numerical evaluation of the method.

Proof. The proof of the theorem is by similar manner as in (Taghizadeh & Mohammadi, 2017).

3. Discrete Formulation

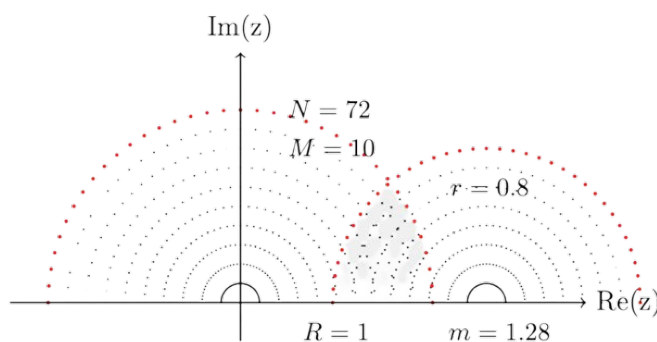


Figura 2. Discretization of the half lens Ω with $M = 10$, $N = 72$, $R = 1$, $r = 0,8$ and $m = 1,28$.

To numerically solve the Dirichlet boundary value problem for the two dimensional Poisson equation in a half lens domain, the domain is first discretized into a mesh of collocation points. A symbolic solution is then computed for a particular problem. The numerical approximation is obtained by evaluating the integral representation derived from the associated Green's function, as presented in the analytical work of Taghizadeh y Mohammadi (2017). At each collocation point, the integrals over the domain are approximated using numerical quadrature through MATLAB's `integral2` function, while boundary integrals are evaluated

using the `integral` function with `waypoints` option to trace the complex contour.

The discretized integral values are assembled into a solution vector, denoted by $\hat{\omega}$, which contains approximations to the solution at each collocation point. This formulation avoids the need to solve a large linear system, as the integral is evaluated directly at each point.

3.1. Domain Discretization

A half lens domain is defined as a mesh of collocation points formed by the points in the interception of the unit disk and a disk centered at m with radius r satisfying $0 < r < 1 < m$, $r^2 + 1 = m^2$ and $z \in \Delta : \text{Im } z > 0$ as illustrated in Figure 2. Considering the discretization of both disk using same values $N \times M$ lattice points with N points in the angular direction and M distinct points in the radial direction, need not to be equidistant.

3.2. Symbolic solution to the problem

Computer algebra system Mathematica was used to implement Algorithm 1 and obtain a symbolic solution to the Dirichlet problem. The script defines the Green's function for the half lens domain, following the formulation by Taghizadeh y Mohammadi (2017), and computes its normal derivative symbolically using the `D` function. Symbolic integrals are evaluated using the `Integrate` function, and the resulting expression is simplified for analytical clarity with the `Simplify` command.

Algorithm 1 Symbolic solution of the Dirichlet problem for the Poisson equation in a half lens domain

- 1: Define the boundary function $\gamma(t)$ ▷ Boundary condition on the half lens
 - 2: Define the source function $f(\zeta)$ ▷ Right-hand side of the Poisson equation
 - 3: Define the Green's function $G_1(z, \zeta)$
 - 4: Compute the integral over the interval $[m - r, 1]$:

$$\text{Integral1} \leftarrow \frac{1}{2\pi i} \int_{m-r}^1 \gamma(t) \left(\frac{(z-\bar{z})}{|t-z|^2} - \frac{(z-\bar{z})}{|t-1|^2} + \frac{r^2(z-\bar{z})}{|t(1-mz)-(m-z)|^2} - \frac{r^2(z-\bar{z})}{|t(m-z)-(1-mz)|^2} \right) dt$$
 - 5: Compute the integral over the boundary of the unit disk Ω_D :

$$\text{Integral2} \leftarrow \frac{1}{2\pi i} \int_{\partial\Omega_D} \frac{\gamma(\zeta)}{\zeta} \left(\frac{(1-|z|^2)}{|1-\zeta z|^2} - \frac{(1-|z|^2)}{|\zeta-z|^2} - \frac{r^2(1-|z|^2)}{|\zeta(1-mz)-(m-z)|^2} + \frac{r^2(1-|z|^2)}{|\zeta(1-mz)-(m-z)|^2} \right) d\zeta$$
 - 6: Compute the integral over the boundary of the smaller domain Ω_{D_m} :

$$\text{Integral3} \leftarrow \frac{1}{2\pi i} \int_{\partial\Omega_{D_m}} \frac{\gamma(\zeta)}{\zeta-m} \left(\frac{r^2|z-m|^2}{|\zeta-z|^2} - \frac{r^2|z-m|^2}{|1-z\zeta|^2} + \frac{r^2|z-m|^2}{|\zeta(1-mz)-(m-z)|^2} - \frac{r^2|z-m|^2}{|\zeta(1-mz)-(m-z)|^2} \right) d\zeta$$
 - 7: Compute the domain integral with Green's function over the half lens domain Ω :

$$\text{Integral4} \leftarrow -\frac{1}{\pi} \iint_{\Omega} f(\zeta) G_1(z, \zeta) d\zeta d\eta$$
 - 8: Combine the contributions to define the solution $\omega(z)$:

$$\hat{\omega}(z) \leftarrow \text{Integral1} + \text{Integral2} + \text{Integral3} + \text{Integral4}$$
-

This symbolic solution $\hat{\omega}(z)$ provides an integral representation function which can serve for numerical evaluation.

3.3. Integral evaluation

According to Harwood y Dupère (2012), when employing the integral equation formulation, one generally requires a fundamental solution to the governing differential equation to construct the overall solution. This fundamental solution also satisfies the boundary conditions, in which case it is referred to as a Green's function. Once available, as demonstrated in the work of Taghizadeh y Mohammadi (2017), it enables the solution of the problem via its integral representation, which is then be evaluated numerically at each collocation point in the domain and along the boundaries.

The numerical integration scheme used to handle the radial direction in the domain is numerical quadrature through MATLAB's functions (Shampine, 2008a, 2008b).

Furthermore, MATLAB's native functions, designed for real-valued integrals, require meticulous handling of complex integrands, typically by separating real and imaginary components, which amplifies computational cost. In this type of integrals challenges arise in managing the high-dimensionality and avoid singularities, especially if the integrand is coupled with multiple complex conjugate terms and absolute values.



4. Numerical results and discussion

In this section, we present numerical experiments computed using Matlab R2022b on a 64-bit Apple M1 chip with 8 GB of RAM, running macOS version 12.3.1.

Some parameters remains constant across all experiments such as: $R = 1,0$, $r = 0,8$, $m = \sqrt{1 + r^2}$, tolerance parameter was set to $1e - 3$, indicating that points within 0,01 units of the boundary are considered boundary points (red) as shown in Figure 3. Other parameters correspond to the half lens domain discretization, refining mesh in angular direction (N) and radial direction (M).

Note that, when solving the problem, different values of N and M were used. The values considered include 64, 128, 256, 1024 and various combinations of these values were analyzed.

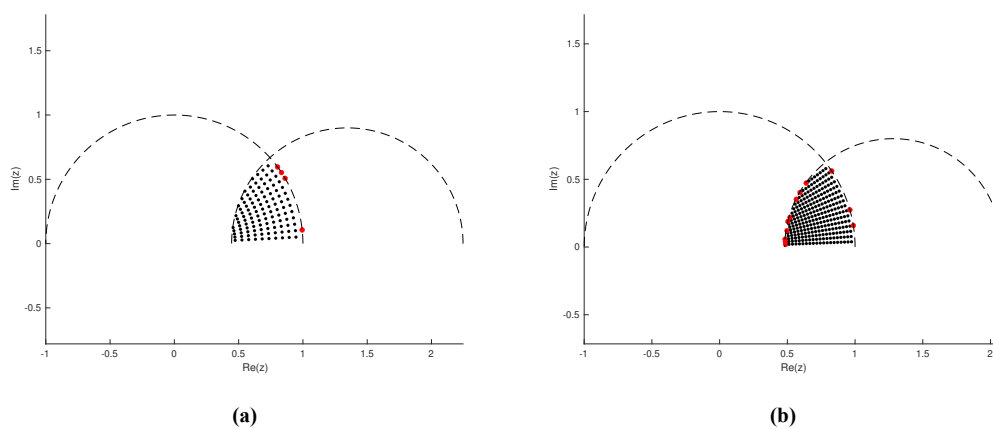


Figura 3. a) Discretization of the half lens without points on the boundary corresponding to the disk with radius r due to the absence of tolerance verification. b) Discretization with tolerance verification using a value of $1e - 3$, including points on all the domain boundary.

Test were conducted in the domain discretization with $N = 64$, $M = 128$ providing a starting reference of the domain refinement process, which improve in a dense and consistent distribution of collocation points along the curved boundary and within the domain with parameters $N = 1024$, $M = 1024$. The concentration of points near the curved boundary is particularly noteworthy, as illustrated in Figure 4.

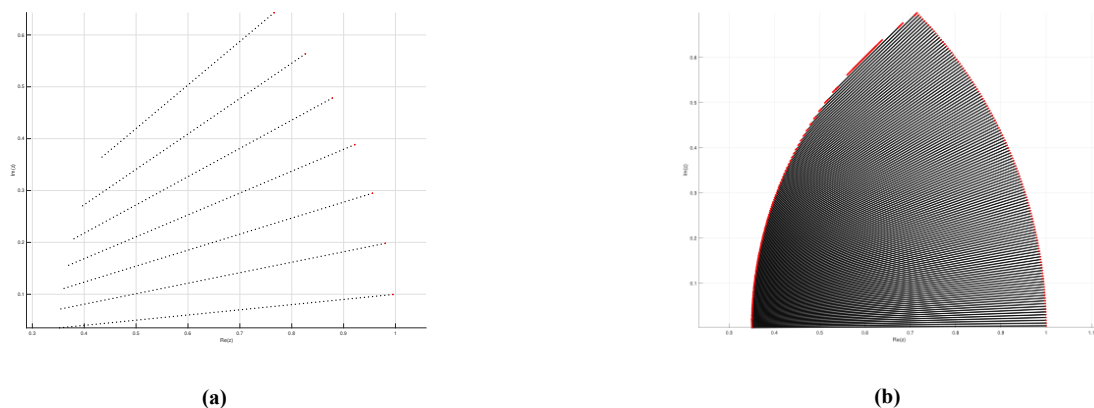


Figura 4. a) Discretization of the half lens parametrized with $N = 64$, $M = 128$ b) Refined domain discretization of the half lens parametrized with $N = 1024$, $M = 1024$

Using collocation points is efficient in the half lens, as they adapt well to the curvature and align closely with the boundary. This approach is advantageous for ensuring that numerical solutions to partial differential equations are accurate in regions of geometric complexity.

4.1. Problem 1

Consider the Dirichlet problem for the Poisson equation:

$$\begin{aligned}\omega_{zz} &= e^z & \text{en } \Omega, \\ \omega &= z^2 & \text{en } \partial\Omega,\end{aligned}\tag{8}$$

Algorithm 1 provides the symbolic calculation of the solution to Problem 1. To optimize computation time of this algorithm, it was performed on a high-performance computer, specifically an Alienware M16 R2 equipped with an Intel Core Ultra 9 processor and 64 GB of RAM.

The particular solution obtained is as follows:

$$\begin{aligned}w(z) = & -\frac{1}{2\pi}i \int_{m-r}^1 \frac{2\zeta^2 \frac{d}{d\zeta} \left(\frac{(z-\bar{\zeta})(-1+m+mz-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta})}{(z-\zeta)(-1+m+mz-\zeta)(m-z+mz-\zeta)(-1+z\zeta)} \right)}{(z-\zeta)^2(-1+m+mz-\zeta)^2(z-m(1+z)+\zeta)^2(-1+z\zeta)^2} \\ & \times [z(-1+m+mz-\zeta) - \bar{\zeta}(z-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta}) \\ & + (-1+m+mz-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta}) \\ & + (-1+m+mz-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta}) \\ & + (-1+m+mz-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta}) \\ & + z(z-\zeta)(-1+m+mz-\zeta)(m-z+mz-\zeta)(-1+z\zeta) \\ & \times (z-\bar{\zeta})(-1+m+mz-\bar{\zeta})(m-z+mz-\bar{\zeta}) \\ & \times \frac{d}{d\zeta}\bar{\zeta} \\ & + (-1+m+mz-\zeta)(-z+\zeta)(z-m(1+z)+\zeta)(1-z\zeta) \\ & \times (-1+m+mz-\bar{\zeta})(-z+\bar{\zeta})(1-z\bar{\zeta})\frac{d}{d\zeta}\bar{\zeta} \\ & - (z-\zeta)(-1+m+mz-\zeta)(m-z+mz-\zeta)(-1+z\zeta) \\ & \times (z-\bar{\zeta})(m-z+mz-\bar{\zeta})(-1+z\bar{\zeta})\frac{d}{d\zeta}\bar{\zeta}] d\zeta.\end{aligned}\tag{9}$$

This function obtained via Mathematica is an integral representation of the solution to the Equation 8. The integral is evaluated pointwise using MATLAB's `integral` function, allowing the numerical computation of $\hat{w}$ on domain (z) and boundary points (ζ) . The integration limits are $[m-r, 1]$, in this case $r = 0,8$, $m = 1,2$, then $m-r = 0,48$.

N / M	64	128	256	512	1024
64	0.94 s	1.10 s	1.71 s	2.25 s	3.92 s
128	1.13 s	1.54 s	3.08 s	3.94 s	10.43 s
256	1.83 s	2.72 s	5.64 s	8.02 s	20.91 s
512	3.23 s	5.40 s	11.47 s	17.81 s	41.48 s
1024	5.76 s	10.97 s	17.07 s	40.01 s	68.17 s

Tabla 1. Execution times in seconds for various values of N and M

Execution times are presented in Table 1 for different combinations of the radial direction M and angular direction N in the evaluation of integral representation in Equation 9 over a half lens domain. From a computational perspective, the cost increases monotonically with M and N , as expected due to the increased number of evaluation points.



Notably, the time growth appears to be roughly quadratic in terms of grid size, aligning with the fact that a finer discretization exponentially increases the number of integral evaluations. Various experiments demonstrate that fixing M and varying N enhances the visualization of the domain, as the angular distribution of points provides greater detail along the curved boundary of the lens-shaped region as Figure 5 suggest for the imaginary part of the solution.

Therefore, the results support the strategy of refining N while keeping M moderate to achieve good results between computational cost and visual fidelity.

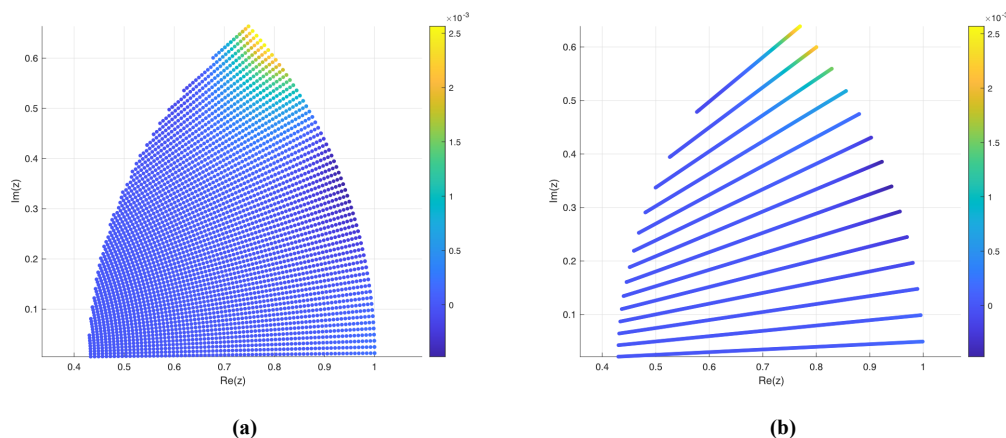


Figura 5. a) Imaginary part of the solution for the parametrized domain with $M=128$ and $N=512$ b) Imaginary part of the solution for the parametrized domain with $M=512$ and $N=128$

To visualize the results, the real part and imaginary part of the solution were generated as shown in Figure 6. Each plot displays the function's values at domain and boundary points using color coded scatter plots over the complex plane. One can verify the smoothness of the solution, its behavior near the boundary, and the overall effectiveness of the integral representation in capturing the properties within the domain.

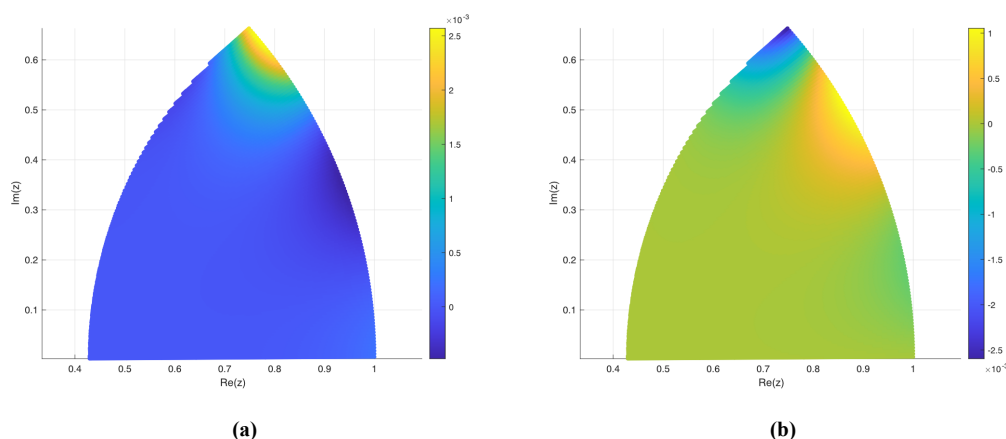


Figura 6. a) Imaginary part of the solution for the parametrized domain with $M=1024$ and $N=1024$ b) Imaginary part of the solution for the parametrized domain with $M=1024$ and $N=1024$

Figure 6 shows plots computed over a parametrized half lens domain with parameters $M=1024$ and $N=1024$. The structure and smooth variation of both parts observed regularity and lack of discontinuities or singularities. Moreover, the function appears to respect boundary conditions regularly.

The results show no signs of numerical instability, with imaginary and real parts exhibiting continuous gradients and physically plausible behavior under the constraints of the problem. The symmetry and boundary conformity of the plots further affirm the reliability of the implementation.

5. Conclusions

A particular solution based on the Green's function and its integral representation is employed in Mathematica to obtain a symbolic formulation for the Dirichlet problem associated with the Poisson equation on a half lens domain. This representation serves as the foundation for implementing a numerical approximation using Matlab integral functions.

The discretization of the domain and its boundary is governed by two parameters, N and M , corresponding to the angular and radial distribution of collocation points, respectively. Numerical experiments and visualizations indicate that maintaining a moderate value for M while refining N yields a favorable balance between computational efficiency and solution accuracy.

Timing results confirm that increased resolution significantly impacts computational time; however, such refinement is essential to capturing the fine structure of the solution. In experiments, setting $N = M = 1024$ offers a good approximation, beyond additional refinement that yields marginal improvement. The computed solution is smooth, visually coherent and consistent.

Future work may involve implementing a fast algorithm to compare to traditional numerical methods and to improve computational efficiency.

6. Referencias

- Begehr, H. & Vaitekhovich, T. (2014). Schwarz problem in lens and lune. *Complex Variables and Elliptic Equations*, 59(1), 76-84.
- Berger, J. M. & Lasher, G. J. (1958). The use of discrete Green's functions in the numerical solution of Poisson's equation. *Illinois Journal of Mathematics*, 2(4A).
- Cedeño, R. & Vanegas, C. (2022). Función de Green vía mapeo conforme para el semiplano superior agrietado. *Matemática-Revista tecnológica ESPOL*, 20.
- Cogollos, S., Taroncher, M. & Boria, V. E. (2009). Efficient and accurate computation of Green's function for the Poisson equation in rectangular waveguides. *Radio Science*, 44(3).
- Harwood, A. R. G. & Dupère, I. D. J. (2012). *Numerical Evaluation of the Compact Green's Function for the Solution of Acoustic Flows*. American Society of Mechanical Engineers.
- Mandelbaum, R. (2018). Weak Lensing for Precision Cosmology. *Annual Review of Astronomy and Astrophysics*, 56(1), 393-433.
- Memarian, M. & Eleftheriades, G. V. (2013). Light concentration using hetero-junctions of anisotropic low permittivity metamaterials. *Light: Science and Applications*, 2(11), e114-e114.
- Shampine, L. (2008a). Matlab program for quadrature in 2D. *Applied Mathematics and Computation*, 202(1), 266-274.
- Shampine, L. (2008b). Vectorized adaptive quadrature in MATLAB. *Journal of Computational and Applied Mathematics*, 211(2), 131-140.
- Shior, M., Agbata, B., Obeng-Denteh, W., Kwabi, P., Ezugorie, I., Marcos, S., Asante-Mensa, F. & Abah, E. (2024). Numerical solution of partial differential equations using MATLAB: Applications to one-dimensional heat and wave equations. *Scientia Africana*, 23(4), 243-254.
- Taghizadeh, N. & Mohammadi, V. S. (2017). Dirichlet and Neumann problems for Poisson equation in half lens. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 52(2), 61-69.
- Vaitsikhovich, T. (2008). *Boundary value problems for complex partial differential equations in a ring domain* (Tesis doctoral).
- Velez Cantos, C. & Vanegas Espinoza, C. (2022). Función de Green en una franja horizontal infinita de amplitud π con frontera mixta Dirichlet-Neumann usando el método parqueting-reflections. *Matemática ESPOL - FCNM Journal*, 20.
- Vergara Ibarra, J. & Vanegas Espinoza, C. (2022). El Kernel de Poisson para un dominio doblemente conexo. *Matemática ESPOL - FCNM Journal*, 20.



7. Contribution of the authors

Author	Contribution
Diego Toala	Article design, manuscript writing, and literature review
Judith Vanegas	Methodology, review, and literature review